

1) (a) $\int_0^{24} R(t) dt = 6(10.4 + 11.2 + 11.3 + 10.2)$
 $= 6(43.1)$ width=6
 $= 258.6$

(b) *yes* since $R(t)$ is differentiable and $R(0) = R(24) = 9.6$, by Rolle's theorem there exists some c $0 < c < 24$ where $R'(c) = 0$.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	→ 10.4
6	10.8
9	→ 11.2
12	11.4
15	→ 11.3
18	10.7
21	→ 10.2
24	9.6

(c) $\frac{1}{24} \int_0^{24} \frac{1}{79} (\pi^2 + 23t - t^2) dt = 10.715$

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$. Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79} (768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

avg rate of water flow = avg value of a function here

2) (a) $\frac{1}{15} \int_0^{15} W(t) dt \approx \frac{3}{2} [20 + 2(31) + 2(28) + 2(24) + 21] = 376.5$

(b) $W'(12) = -0.549$ °C/day *on 12th day temperature is dropping at about -0.549 °C/day*

(c) $W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{21 - 24}{6} = -\frac{3}{6} = -\frac{1}{2}$ °C/day

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

(d) $\frac{1}{15} \int_0^{15} P(t) dt = 25.757$

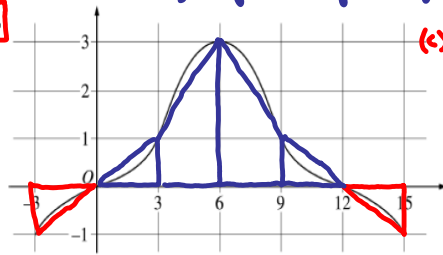
The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.

- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- (c) A student proposes the function P , given by $P(t) = 20 + 10te^{-t/3}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

3)

(a)
 $g(x) = 5 + \int_6^x f(t) dt = 5$
 $g'(x) = f(x)$
 $g'(6) = f(6) = 3$
 $g''(6) = f'(6) = 0$

(b) g is decreasing when $g'(x) < 0$
 $g'(x) = f(x) < 0$ for $[-3, 0) \cup (12, 15]$



(c) g concave down when $g''(x) < 0$ or $f'(x) < 0$
 i.e. $6 < x < 15$

Graph of f

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t) dt$ for $-3 \leq x \leq 15$.

- (a) Find $g(6)$, $g'(6)$, and $g''(6)$.
- (b) On what intervals is g decreasing? Justify your answer.
- (c) On what intervals is the graph of g concave down? Justify your answer.
- (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.

(d) $\int_{-3}^{15} f(t) dt = 2 \left[\frac{3}{2}(1+3) \right] = 12$

4)

(a) $\int_0^{40} v(t) dt$
 $= 10(9.2 + 7 + 2.4 + 4.3)$
 $= 10(22.9) = 229 \text{ miles}$
 distance traveled in first 24 hrs.

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(b) Since $v(0) = v(15)$, Rolle's Theorem guarantees $a(t) = 0$ on $0 < t < 15$ likewise on $25 < t < 30$

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

since $v(15) = v(30)$

(c)
 $a(23)$
 $= -0.408$
 miles/min²

(d) $\frac{1}{40} \int_0^{40} v(t) dt = 5.916$

5)

$$(b) \frac{1}{8} \int_0^8 T(x) dx = \frac{1}{8} \left[\frac{1}{2}(193) + \frac{4}{2}(163) + \frac{1}{2}(132) + \frac{2}{2}(117) \right] = 76.688 \text{ } ^\circ\text{C}$$

$$(a) T'(7) \approx \frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -\frac{7}{2} \text{ } ^\circ\text{C/cm}$$

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^\circ\text{C}$)	100	93	70	62	55

$$(c) \int_0^8 T'(x) dx = T(8) - T(0) = 55 - 100 = -45 \text{ } ^\circ\text{C}$$

this is the difference in temp from one end of the wire to the other

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^\circ\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(d) if $T''(x) > 0$ means concave up. In this case slopes are becoming more and more positive as x increases.

$\frac{93-100}{1-0} = -7$ $\frac{70-93}{5-1} = \frac{-23}{4} = -5.75$ $\frac{62-70}{1} = -8$ Not ok!

Data are not consistent with that assertion!!